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EXPANSION OF A FINITE MASS OF GAS INTO VACUUM

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February 1965

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Some years ago, the authors studied the problem of the expansion into vacuum of a finite mass of gas initially at rest and in a uniform state, under the assumptions that the gas is perfect and inviscid. The results for the case of plane flow appear in Ref. 1, while those for cylindrical and spherical flow are unpublished. From time to time we have, upon request, privately communicated some of the results, together with assurances of the eventual publication of the results in their entirety. However, the continued passage of time has tended to diminish the conviction of these assurances. It therefore seems advisable, at this time, at least to summarize the contents of Ref. 1, and to provide perhaps the most useful result of our unpublished calculations.

We considered the three cases of plane, cylindrical, and spherical symmetry. The flow in the plane case can be described[2] as the interaction of two expansion fans, or simple waves, centered about the edges ($\pm x_o$) of the initial mass of gas. The (x,t)-plane, or the flow at any time t, is thus divided into two essential regimes, a simple wave and an interaction region. In the cylindrical and spherical cases, the regions of the (r,t)-plane are essentially the same, although pure simple waves no longer exist. In all cases, the flow at any point consists first of the outward motion caused by the expansion fan from the nearest boundary, and then a weakening of this process by the arrival of the expansion fan from the other boundary (plane case) or a reflection from the center (cylindrical and spherical cases).

In the case of plane flow, we constructed, by standard methods,[2] an exact analytic solution valid for all t. The cylindrical and spherical cases, however, are not amenable to such a treatment. However, since all of the gas eventually enters the interaction region, a solution, asymptotic for large t, in the interaction region provides useful information about the flow. It is precisely such a solution which we constructed by similarity methods.

The principal results of the analysis are:

1) the flow velocity $u(r,t)$ is, in all three cases,

$$u(r,t) \cong \frac{r}{t} , \qquad (1)$$

where r is the distance from the origin, and

2) $\quad \frac{\rho(r,t)}{\rho_o} \cong c_m(\gamma) \left(\frac{r_o}{a_o t}\right)^m \qquad (2)$

$$m = 1 \text{ plane case}$$
$$= 2 \text{ cylindrical case}$$
$$= 3 \text{ spherical case}$$

where ρ_o is the density, a_o the sound speed, and r_o the radius of the initially uniform gas. The constant in Eq. (2) depends only on m and on the gas constant γ. This solution is asymptotically valid in the region of the (r,t)-plane not too close to the expansion front.

As we mentioned previously, in the case of plane flow an exact analytic solution of the problem can be obtained. By examining this solution in the limit of large t, we were able not only to verify that the assumed similarity solution is in fact the asymptotic flow, but we obtained at the same time an analytic expression for the constant $c_1(\gamma)$, viz.,

$$c_1(\gamma) = 2^{\lambda-1} \frac{[\Gamma(2\nu-1)]^{\lambda}}{[\Gamma(\nu)]^{2\lambda}} \qquad (3)$$

where $\lambda = \frac{(\gamma-1)}{2}$ and $\nu = \frac{1}{2}\frac{(\gamma+1)}{(\gamma-1)}$. In Eq. (3), Γ denotes the usual Gamma-function. In the cylindrical and spherical cases, the

asymptotic validity of the similarity solution was established by a numerical integration of the equations of motion. Integration of the equations for different values of γ served to determine the constants C_2 and C_3 as functions of γ. The dependence of C_m on γ for the various cases is shown in the figure.

The results of this study have practical application as an approximation to the expansion of a finite mass of gas into a gas at much lower pressure and density. In this case, a shock wave precedes the expanding gas, but in the limit of ambient vacuum the effect of the shock wave disappears from the problem. Another application follows from the use of the unsteady analogy of hypersonic small-deflection theory, whereby the cylindrical unsteady flow becomes analogous to the flow of a high Mach number jet to vacuum.
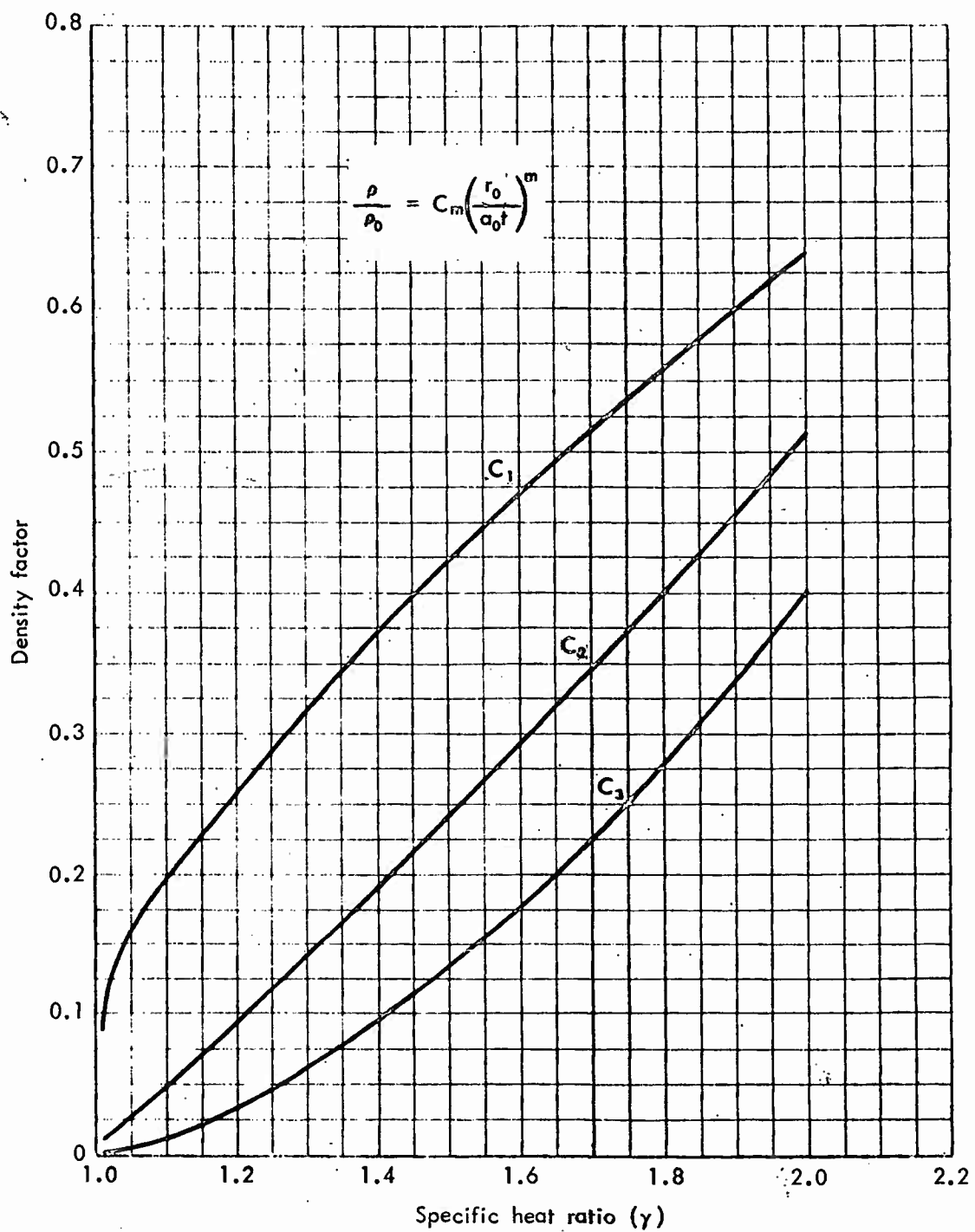
For Journal Article, See AIAAJ. $\overline{3}$, 1200-1201(1965)

and also:

Erratum: AIAAJ $\overline{3}$, P. 2176 (nov-1965)

where equation (3) is given as:

$$C_1(\gamma) = 2^{\left(1-\frac{1}{\gamma}\right)}\left[\Gamma(2\gamma-1)/\Gamma(\gamma)^2\right]$$

$$\frac{\rho}{\rho_0} = C_m\left(\frac{r_0'}{a_0 t}\right)^m$$

REFERENCES

1. Greifinger, C. and J. D. Cole, "One Dimensional Expansion of a Finite Mass of Gas into Vacuum," The RAND Corporation, P-2008, June 1960.   *(For Plane Flow)*.

2. Courant and Friedrichs, "Supersonic Flow and Shock Waves," Interscience Publishers, Inc., New York, pp. 88-91 and pp. 191-197.

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